Approximate Analytical Solutions of Nonlinear Fractional PDEs via the Conformable Laplace Adomian Decomposition Method

Anant S. Patel
Research Scholar,
Department of Mathematics,
Veer Narmad South Gujarat University, Surat, Gujarat, India.
Email:- aspatel@vnsgu.ac.in

Priti V. Tandel
Assistant Professor,
Department of Mathematics,
Veer Narmad South Gujarat University, Surat, Gujarat, India.
Email:- pvtandel@vnsgu.ac.in

Manan A. Maisuria
Temporary Assistant Professor,
Department of Mathematics,
Veer Narmad South Gujarat University, Surat, Gujarat, India.
Email:- mananmaisuria.maths21@vnsgu.ac.in

Abstract:

This study presents the Conformable Laplace Adomian Decomposition Method (CLADM), an analytical framework that integrates the conformable fractional derivative, the conformable Laplace transform, and Adomian polynomial decomposition to derive rapidly convergent series solutions for nonlinear fractional partial differential equations (FPDEs). The proposed formulation eliminates the need for linearization, perturbation techniques, or restrictive simplifying assumptions, thereby preserving the intrinsic nonlinear characteristics of the governing equations. The accuracy and computational performance of CLADM are demonstrated through its application to a conformable fractional damped Burgers equation and a conformable fractional reaction–diffusion equation. Numerical investigations indicate that the proposed method attains absolute errors of the order of 10^{-9} for the damped Burgers model, whereas errors are reduced to approximately 10^{-15} for the reaction–diffusion model over the fractional orders considered, confirming the high accuracy, robustness, and effectiveness of the proposed analytical procedure. These results demonstrate that CLADM provides a highly accurate and robust analytical framework for nonlinear FPDEs, with significant potential for applications to models in materials science, anomalous diffusion, and related complex transport phenomena.

1. Introduction:

Nonlinearity pervades nearly all physical phenomena, prompting intense scientific and engineering focus on nonlinear equations over the past decade. The mathematical modelling of nonlinear systems with memory-dependent behaviour has stimulated extensive interest in fractional-order partial differential equations (FPDEs). By naturally incorporating hereditary effects into the governing equations, FPDEs provide a powerful framework for analysing complex dynamical processes across diverse scientific and engineering applications. Owing to these distinctive characteristics, FPDEs have become fundamental mathematical models in numerous scientific and engineering disciplines. The flexibility of the FPDE framework has enabled its adoption across a broad spectrum of theoretical and applied disciplines. Representative applications include quantum mechanics, optical sciences, superconductivity, biological and chemical systems, fluid mechanics, electromagnetism, vibration and acoustic analysis, signal processing, and the characterization of polymer-based materials (Abassy et al., 2007; Ait Touchent et al., 2018; Javeed et al., 2018; Jothimani et al., 2019). The extraordinary prevalence of FPDEs across multiple engineering and scientific disciplines reflects their superior capacity to capture complex physical phenomena compared to classical integer-order equations, motivating substantial theoretical and applied research contributions (Khan et al., 2020; Moallem Ganji-Hossein Jafari, 2020; Shah et al., 2019; Shah, Farooq, et al., 2020; Shah, Khan, et al., 2020).

The mathematical foundation of fractional calculus extends back nearly three centuries. Joseph Liouville (1832–1837) established rigorous mathematical frameworks for fractional integration, while Bernhard Riemann's subsequent contributions developed the Riemann–Liouville integral, which served as the primary foundation for fractional calculus for nearly two centuries. The Caputo fractional derivative, introduced by Michele Caputo in 1967, represented a significant advancement by providing a derivative definition particularly well-suited for initial value problems in applied mathematics and physics. Despite these developments, classical fractional derivatives—particularly Riemann–Liouville and Caputo formulations—possessed limitations: they did not uniformly preserve classical calculus properties, exhibited counterintuitive behaviours for simple functions, and raised interpretive challenges in physical applications.

To overcome the mathematical and computational challenges associated with classical fractional derivatives, the conformable fractional derivative was introduced as an alternative fractional operator. Its rigorous formulation, further refined by Abdeljawad, maintains many of the core properties of classical differentiation, enabling a more natural extension of standard calculus concepts to the fractional-order setting. These include the validity of the chain rule, the existence of exponential-type solutions, the formulation of the Laplace transform, integration by parts, Taylor series expansions, and a fractional analogue of Gronwall's inequality, thereby providing a mathematically consistent framework for the analysis of fractional differential equations (Abdeljawad, 2015). Building on this foundation, Atangana characterized essential properties of conformable derivatives, demonstrating their mathematical elegance and physical interpretability (Atangana et al., 2015). The conformable derivative's ability to preserve classical calculus structures—linearity, product rule, quotient rule, power rule, and chain rule—while maintaining mathematical rigor offers substantial advantages over previous formulations.

Practical applications of conformable fractional derivatives have proliferated rapidly across numerous domains. Cenesiz and Kurt (Çenesiz & Kurt, 2015) solved heat differential equations in time-space fractional form using conformable derivatives, while Kurt (Kurt et al., 2015) developed analytical approximations of the time-conformable Burgers' equation via the homotopy analysis method (HAM). Chung's research on conformable derivatives and integrals extended the theoretical framework to fractional Newtonian mechanics (Chung, 2015), and

Ekici's investigations addressed optical solitons with fractional temporal evolution under Hamiltonian perturbations (Ekici et al., 2016). The conformable extension of the classical Nikiforov–Uvarov method provides an efficient analytical framework for deriving spectral solutions of conformable fractional differential equations. Furthermore, the existence and uniqueness of solutions have been established for a class of conformable fractional differential equations, reinforcing the mathematical foundation of the conformable fractional calculus (Karayer et al., 2016; Gökdogan et al., 2016). Cenesiz (Çenesiz et al., 2016) discovered exact analytical solutions to conformable Burgers'-type equations, and Unal and Gokdogan (Ünal & Gökdoğan, 2017) developed the conformable differential transform method for ordinary differential equations. These diverse contributions—encompassing Bayram, Mozaffari, Hosseini, Zhao, Kaplan, and numerous other researchers—collectively demonstrate the conformable derivative's efficacy across varied problem classes (Acan et al., 2018; Bayram et al., 2018; Hosseini, Bekir, et al., 2017; Hosseini, Mayeli, et al., 2017; Kaplan et al., 2017; Mozaffari et al., 2018; Zhao & Luo, 2017).

Despite substantial progress in fractional calculus theory and methods, critical research gaps persist. While individual components—conformable derivatives, Laplace transforms, and Adomian decomposition—have been extensively studied independently, their systematic integration into a unified, rigorous framework for solving general nonlinear fractional PDEs remains underdeveloped. The Laplace–Adomian Decomposition Method (LADM), which strategically combines Laplace transform advantages with ADM's decomposition framework, achieves rapid convergence and handles dispersive phenomena arising in plasma physics and quantum mechanics. The conformable Laplace Adomian decomposition method (CLADM) proposed in this work directly addresses these research gaps by establishing a comprehensive analytical framework combining modern conformable fractional calculus with proven decomposition methodologies.

The structure of this paper is outlined as follows. Section 2 introduces the essential concepts of conformable fractional calculus, including the conformable fractional derivative, the associated conformable Laplace transform, and the convergence theorem used in the subsequent analysis. Section 3 develops the Conformable Laplace Adomian Decomposition Method (CLADM) for a general class of nonlinear conformable fractional partial differential equations. In Section 4, the proposed method is applied to two representative test problems, and the resulting numerical solutions are analyzed through tables and graphical illustrations to assess the accuracy and effectiveness of the method. Finally, Section 5 concludes the paper by summarizing the principal findings and highlighting the significance of the proposed approach.

2. Basics of Conformable Derivatives

Definition 1:

For a function, $\Omega:[0,\infty)\rightarrow\square$, the conformable fractional derivative $\tilde{n}\in(0,1]$ at $\tau>0$ is defined by (Khalil et al., 2014)

$$\left(\mathsf{T}_{\tilde{n}}\Omega\right)(\tau)=\lim_{\grave{o}\to 0}\frac{\Omega(\tau+\grave{o}\tau^{1-\tilde{n}})-\Omega(\tau)}{\grave{o}} \tag{1.1}$$

For all $\tau>0$.

Definition 3:

Let $\tilde{n}\in(0,1]$ and $\Omega:[0,\infty)\rightarrow\square$ be real valued function. Then, the Laplace transform for fractional order $\tilde{n}$ is defined by (Abdeljawad, 2015; Silva et al., 2018)

$$\mathsf{L}_{\tilde{n}}[\Omega(\tau)](s)=\int_0^\infty E_{\tilde{n}}(-s,\tau)\Omega(\tau)\,d_{\tilde{n}}\tau \tag{1.2}$$

where $E_{\tilde{n}}(-s,\tau)=exp\left(-s\frac{\tau^{\tilde{n}}}{\tilde{n}}\right)$

The Laplace transform in the conformable fractional-order sense is described as

$$\mathsf{L}_{\tilde{n}}[\mathsf{D}_\tau^{(\tilde{n})}\Omega(\tau)]=s\mathsf{L}_{\tilde{n}}[\Omega(\tau)]-\Omega(0) \tag{1.3}$$

The usual and fractional Laplace transform has the following relation.

Theorem 1:

Let $\Omega:[0,\infty)\rightarrow\square$ be a function, and if $\mathsf{L}_{\tilde{n}}\Omega(\tau)(s)=\Theta\tilde{n}(s)$ exists, then

$$\Theta_{\tilde{n}}(s)=\mathrm{L}\left[\Omega\left((\tilde{n}\tau)^{1/\tilde{n}}\right)\right](s) \tag{1.4}$$

where $\mathrm{L}[G(\tau)](s)=\int_0^\infty e^{-s\tau}G(\tau)\,d\tau$. (Silva et al., 2018)

Theorem 2:

If $\Omega(\xi,\tau)=\sum_{k=0}^{\infty}\Phi_k(\xi)\tau^k$ is a given series (Tandel et al., 2025);

1. If $\exists\ 0<\sigma<1$ such that $\frac{\|\Phi_{k+1}\|}{\|\Phi_k\|}\leq\sigma$, then the given series solution is convergent.

2. If $\exists\ \sigma>1$ such that $\frac{\|\Phi_{k+1}\|}{\|\Phi_k\|}\geq\sigma$, then the given series solution is divergent.

Consider the following FPDEs forms:

$$\mathsf{D}_\tau^{\tilde{n}}\Omega(\xi,\tau)+\mathsf{D}_\xi^{n}\Omega(\xi,\tau)+\mathrm{R}\left(\Omega(\xi,\tau)\right)+\mathrm{N}\left(\Omega(\xi,\tau)\right)=g(\xi,\tau) \tag{1.5}$$

where $\xi>0,\ \tau>0,\ 0<\tilde{n}\leq 1$

The fractional derivative $\mathrm{D}_{\tau}^{\tilde{n}}$ in equation (1.5) is considered in the conformable sense in τ, D_{ξ}^{n} is the highest-order classical derivative operator in ξ, R is other lower-order derivative terms, N is the nonlinear term, and $g(\xi,\tau)$ is the sources term with the initial condition

$$\Omega(\xi,0) = f(\xi), \qquad (1.6)$$

Applying the conformable Laplace transform $\mathrm{L}_{\tilde{n}}$ with respect to τ both sides of equation(1.5), we have

$$\mathrm{L}_{\tilde{n}}[\mathrm{D}_{\tau}^{\tilde{n}}\Omega] + \mathrm{L}_{\tilde{n}}[\mathrm{D}_{\xi}^{n}\Omega] + \mathrm{L}_{\tilde{n}}[\mathrm{R}\,(\Omega) + \mathrm{N}\,(\Omega)] = \mathrm{L}_{\tilde{n}}[g(\xi,\tau)], \qquad (1.7)$$

By applying property equation (1.3) of conformable Laplace transform, equation (1.7) becomes

$$s\mathrm{L}_{\tilde{n}}[\Omega] - \Omega(\xi,0) + \mathrm{L}_{\tilde{n}}[\mathrm{D}_{\xi}^{n}\Omega] + \mathrm{L}_{\tilde{n}}[\mathrm{R}\,(\Omega) + \mathrm{N}\,(\Omega)] = \mathrm{L}_{\tilde{n}}[g(\xi,\tau)], \qquad (1.8)$$

Which can be rewritten as

$$\mathrm{L}_{\tilde{n}}[\Omega] = \frac{1}{s}\left[\Omega(\xi,0) + \mathrm{L}_{\tilde{n}}\left\{g(\xi,\tau)\right\}\right] - \frac{1}{s}\mathrm{L}_{\tilde{n}}[\mathrm{D}_{\xi}^{n}\Omega] - \frac{1}{s}\mathrm{L}_{\tilde{n}}[\mathrm{R}\,(\Omega) + \mathrm{N}\,(\Omega)], \qquad (1.9)$$

where the nonlinear term $\mathrm{N}\,(\Omega)$ of equation (1.5) is handled by applying ADM and the solution is expressed as an infinite sum of series:

$$\Omega(\xi,\tau) = \sum_{k=0}^{\infty}\Omega_k(\xi,\tau), \qquad (1.10)$$

$$\mathrm{N}\,\left(\Omega(\xi,\tau)\right) = \sum_{k=0}^{\infty}A_k, \qquad (1.11)$$

where, Adomian polynomials A_k are given by,

$$A_k = \frac{1}{k!}\frac{d^k}{d\lambda^k}\left[\mathrm{N}\left(\sum_{j=0}^{k}\lambda^j\Omega_j\right)\right]\Big|_{\lambda=0}, \quad k = 0,1,2,3,\ldots \qquad (1.12)$$

By substituting equations (1.10) and (1.11) into equation (1.9)

$$\begin{aligned}\sum_{k=0}^{\infty}\Omega_k &= \left[\frac{1}{s}\left[\Omega(\xi,0) + \mathrm{L}_{\tilde{n}}\left\{g(\xi,\tau)\right\}\right]\right] - \left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\sum_{k=0}^{\infty}\mathrm{D}_{\xi}^{n}\Omega_k\right\}\right] \\ &\quad - \left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{R}\left(\sum_{k=0}^{\infty}\Omega_k\right) + \sum_{k=0}^{\infty}A_k\right\}\right],\end{aligned} \qquad (1.13)$$

In equation(1.13), applying inverse Laplace transform in conformable sense, we have

$$\sum_{k=0}^{\infty}\Omega_k = \mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\left[\Omega(\xi,0)+\mathrm{L}_{\tilde{n}}\left\{g(\xi,\tau)\right\}\right]\right]-\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\sum_{k=0}^{\infty}\mathrm{D}_{\xi}^{n}\Omega_k\right\}\right] \\ -\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{R}\left(\sum_{k=0}^{\infty}\Omega_k\right)+\sum_{k=0}^{\infty}A_k\right\}\right], \tag{1.14}$$

When comparing both sides of equation(1.14), we get the following recursive relation:

$$\Omega_0 = \mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\left[\Omega(\xi,0)+\mathrm{L}_{\tilde{n}}\left\{g(\xi,\tau)\right\}\right]\right], \tag{1.15}$$

$$\Omega_1 = -\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{D}_{\xi}^{n}\Omega_0\right\}\right]-\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{R}\left(\Omega_0\right)+A_0\right\}\right], \tag{1.16}$$

$$\Omega_2 = -\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{D}_{\xi}^{n}\Omega_1\right\}\right]-\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{R}\left(\Omega_1\right)+A_1\right\}\right], \tag{1.17}$$

In general, the relation can be written as

$$\Omega_{k+1} = -\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{D}_{\xi}^{n}\Omega_k\right\}\right]-\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\mathrm{R}\left(\Omega_k\right)+A_k\right\}\right] \tag{1.18}$$

where $k = 0,1,2,3,\dots$ with Ω_0 as equation (1.15).

The solution $\Omega(\xi,\tau)$ can be obtained from equation(1.10).

3. Application of CLADM

Problem 1:

Considering the following fractional Damped Burger equation (Acan et al., 2018)

$$\mathrm{D}_{\tau}^{\tilde{n}}\Omega(\xi,\tau)+\Omega(\xi,\tau)\frac{\partial\Omega(\xi,\tau)}{\partial\xi}+\frac{\partial^2\Omega(\xi,\tau)}{\partial\xi^2}+\frac{1}{5}\Omega(\xi,\tau)=0 \tag{2.1}$$

subject to the initial condition

$$\Omega(\xi,0)=\frac{1}{5}\xi \tag{2.2}$$

Solution:

By applying CLADM in equation(2.1), we get

$$\sum_{k=0}^{\infty}\Omega_k = \mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\Omega(\xi,0)\right]-\mathrm{L}_{\tilde{n}}^{-1}\left[\frac{1}{s}\mathrm{L}_{\tilde{n}}\left\{\sum_{k=0}^{\infty}A_k+\sum_{k=0}^{\infty}\left(\frac{\partial^2\Omega_k}{\partial\xi^2}\right)+\frac{1}{5}\sum_{k=0}^{\infty}\Omega_k\right\}\right] \tag{2.3}$$

From the initial condition equation(2.2), we can write

$$\Omega_0(\xi,\tau)=\frac{\xi}{5} \tag{2.4}$$

by substituting equation (2.4) with equation (2.3), we obtain

$$\Omega_1(\xi,\tau) = -\frac{2\tau^{\tilde{n}}\xi}{25\tilde{n}}$$

$$\Omega_2(\xi,\tau) = \frac{3\tau^{2\tilde{n}}\xi}{125\tilde{n}^2}$$

$$\Omega_3(\xi,\tau) = -\frac{13\tau^{3\tilde{n}}\xi}{1875\tilde{n}^3} \tag{2.5}$$

$$\Omega_4(\xi,\tau) = \frac{\tau^{4\tilde{n}}\xi}{500\tilde{n}^4}$$

The final series solution is then given by

$$\Omega(\xi,\tau) = \frac{\xi}{5} - \frac{2\tau^{\tilde{n}}\xi}{25\tilde{n}} + \frac{3\tau^{2\tilde{n}}\xi}{125\tilde{n}^2} - \frac{13\tau^{3\tilde{n}}\xi}{1875\tilde{n}^3} + \frac{\tau^{4\tilde{n}}\xi}{500\tilde{n}^4} \tag{2.6}$$

For $\tilde{n} = 1$, the exact solution of equation (2.1) is given as:

$$\Omega(\xi,\tau) = \frac{\xi}{5\left(2e^{\frac{\tau}{5}} - 1\right)} \tag{2.7}$$

Problem 2:

Now we consider the following FPDE (Acan et al., 2018):

$$D_\tau^{\tilde{n}}\Omega(\xi,\tau) - \frac{\partial\Omega(\xi,\tau)}{\partial\xi}\frac{\partial^2\Omega(\xi,\tau)}{\partial\xi^2} - \frac{\partial^2\Omega(\xi,\tau)}{\partial\xi^2} + \Omega(\xi,\tau)\frac{\partial^3\Omega(\xi,\tau)}{\partial\xi^3} = 0 \tag{2.8}$$

subject to the initial condition

$$\Omega(\xi,0) = \frac{e^{\frac{\xi}{4}}}{4} \tag{2.9}$$

Solution:

By applying CLADM in equation(2.8), we get

$$\sum_{k=0}^{\infty}\Omega_k = L_{\tilde{n}}^{-1}\left[\frac{1}{s}\Omega(\xi,0)\right] - L_{\tilde{n}}^{-1}\left[\frac{1}{s}L_{\tilde{n}}\left\{\sum_{k=0}^{\infty}A_k + \sum_{k=0}^{\infty}\left(\frac{\partial^2\Omega_k}{\partial\xi^2}\right) - \sum_{k=0}^{\infty}B_k\right\}\right] \tag{2.10}$$

From the initial condition equation(2.9), we can write

$$\Omega_0(\xi,\tau) = \frac{e^{\frac{\xi}{4}}}{4} \tag{2.11}$$

by substituting equation (2.11) with equation (2.10), we obtain

$$\Omega_1(\xi,\tau) = \frac{\tau^{\tilde{n}}\, e^{\frac{\xi}{4}}}{64\tilde{n}}$$

$$\Omega_2(\xi,\tau) = \frac{\tau^{2\tilde{n}}\, e^{\frac{\xi}{4}}}{2048\tilde{n}^2} \tag{2.12}$$

$$\Omega_3(\xi,\tau) = \frac{\tau^{3\tilde{n}}\, e^{\frac{\xi}{4}}}{98304\tilde{n}^3}$$

$$\Omega_4(\xi,\tau) = \frac{\tau^{4\tilde{n}}\, e^{\frac{\xi}{4}}}{6291456\tilde{n}^4}$$

The final series solution is then given by

$$\Omega(\xi,\tau) = \frac{e^{\frac{\xi}{4}}}{4} + \frac{\tau^{\tilde{n}}\, e^{\frac{\xi}{4}}}{64\tilde{n}} + \frac{\tau^{2\tilde{n}}\, e^{\frac{\xi}{4}}}{2048\tilde{n}^2} + \frac{\tau^{3\tilde{n}}\, e^{\frac{\xi}{4}}}{98304\tilde{n}^3} + \frac{\tau^{4\tilde{n}}\, e^{\frac{\xi}{4}}}{6291456\tilde{n}^4} \tag{2.13}$$

For $\tilde{n} = 1$, the exact solution of equation (2.10) is given as:

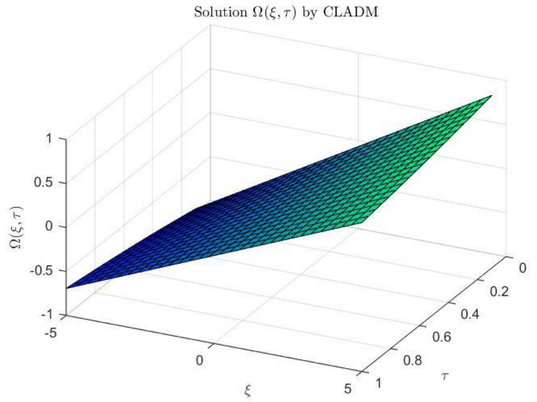
$$\Omega(\xi,\tau) = \frac{1}{4} e^{\frac{1}{4}\left(\xi + \frac{\tau}{4}\right)} \tag{2.14}$$

4. Results and Discussion:

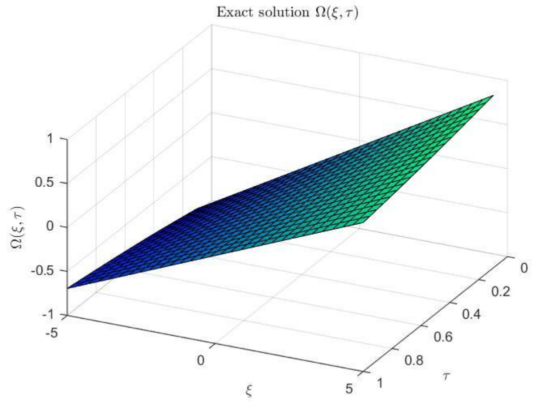
The Conformable Laplace Adomian Decomposition Method was validated on two nonlinear fractional PDEs by comparing numerical solutions with exact solutions at $\tilde{n} = 1$. For Problem-1 (fractional Damped Burgers equation), Table 4.1 shows absolute errors of 1.01×10^{-9} to 5.04×10^{-9} at $\tau = 0.2$ across all spatial points $\xi \in [0.1, 0.5]$. At longer time $\tau = 0.5$, errors increase to 2.27×10^{-7} to 1.14×10^{-6} indicating that truncation error accumulates over time in this nonlinear problem. Figure 4.1 displays three-dimensional surfaces comparing CLADM and exact solutions, showing near-perfect agreement. The solution behavior at varying fractional orders ($\tilde{n}$ = 0.3, 0.6, 0.8, 1.0) is presented in Figure 4.2, where at $\tau = 0.2$ (Figure 4.2a) fractional order effects are modest, but at $\tau = 0.8$ (Figure 4.2b) the differences become more pronounced, reflecting accumulated memory effects. Figure 4.3 visualizes the absolute error distribution, showing systematic growth with increasing time and spatial distance, with maximum errors at $\xi = 0.5$.

Problem 2 (fractional reaction-diffusion equation) demonstrates superior numerical performance. Table 4.2 reveals absolute errors at machine precision levels (~10^{-15}) at $\tau = 0.2$, ranging from 1.39×10^{-15} to 1.50×10^{-15}, At $\tau = 0.5$, errors remain bounded at 3.33×10^{-13} to 3.68×10^{-13}. Figure 4.4 shows CLADM and exact solution surfaces that are visually indistinguishable. Figure 4.5 displays solution variations across fractional orders with smooth behaviour similar to Problem 1 but with substantially smaller error magnitudes. Figure 4.6 illustrates error at machine precision levels, appearing nearly uniform across the domain without the systematic growth pattern observed in Problem 1. This indicates that Problem 2's mathematical structure permits more stable numerical evolution than Problem 1.

Comparing Tables 4.1 and 4.2 reveals a critical difference: Problem 1 exhibits error growth proportional to temporal evolution (increasing by ~100 times from $\tau = 0.2$ to $\tau = 0.5$), while Problem 2 shows bounded error accumulation with only ~10^3 increase over the same time interval. Both problems satisfy the convergence criterion from Theorem 2, confirmed by the small magnitude of errors across all evaluation points. At the classical order $\tilde{n} = 1$, both problems recover exact solutions with high precision, validating the method's mathematical foundation. The fractional order dependence visible in Figures 4.2 and 4.5 confirms that solutions vary smoothly across α without discontinuities or instabilities, demonstrating the robustness of the conformable derivative framework across the fractional order range.

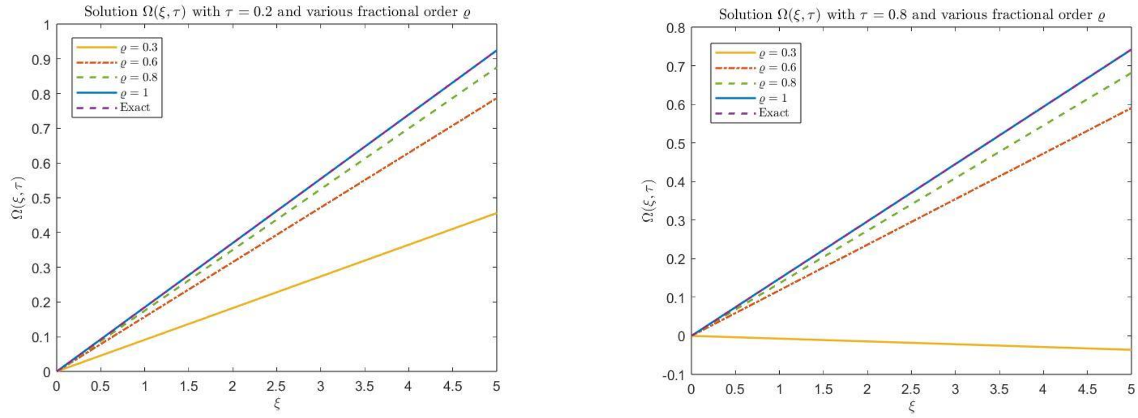


(a)



(b)

FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..1 COMPARISON OF SOLUTION $\Omega(\xi,\tau)$ FOR EXAMPLE-1



(a) (b)

FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..**2 SOLUTION $\Omega(\xi,\tau)$ FOR EXAMPLE-1 WITH VARIOUS**

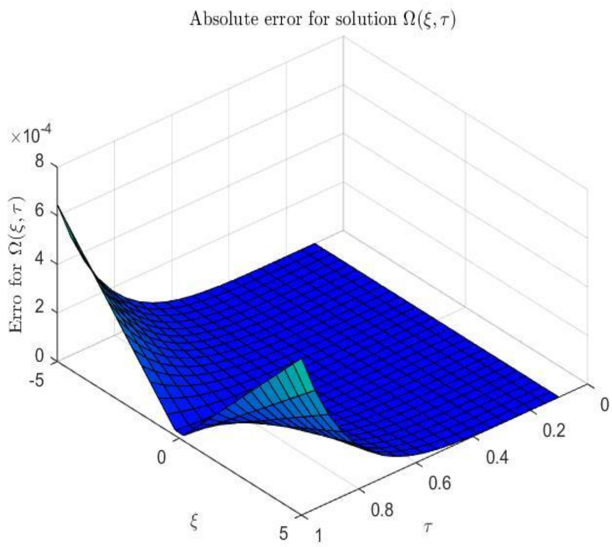


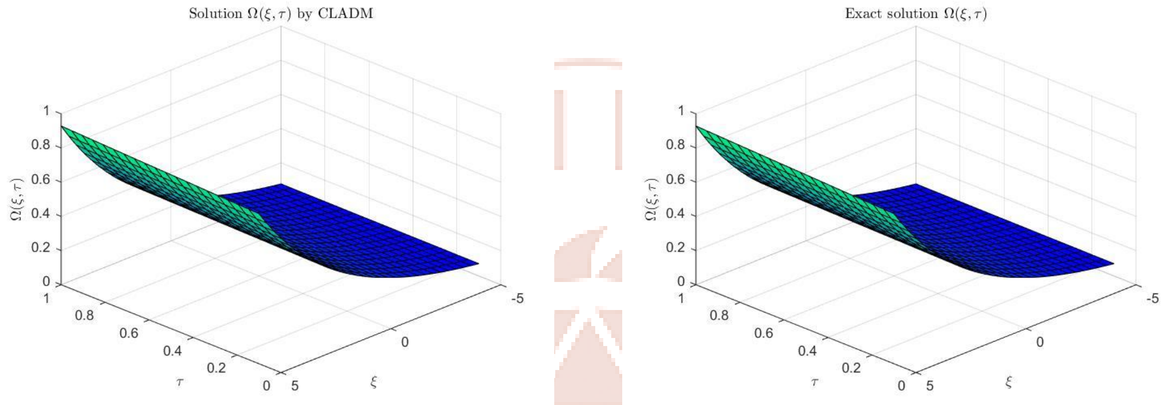
FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..**3**
ABSOLUTE ERROR OF SOLUTION $\Omega(\xi,\tau)$ FOR EXAMPLE-1

TABLE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..**1 NUMERICAL COMPARISON OF $\Omega(\xi,\tau)$ FOR EXAMPLE-1**

WITH VARIOUS FRACTIONAL ORDER ϱ

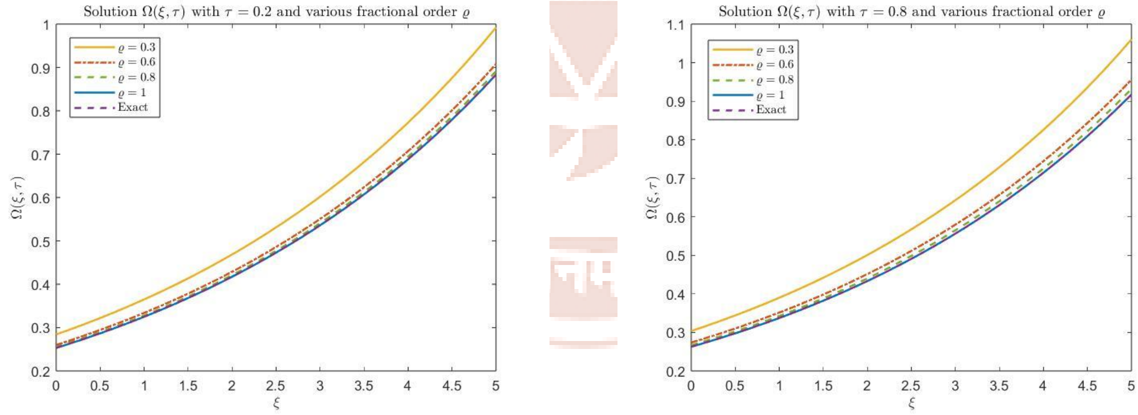
ξ	τ	CLADM				Exact	Abs error
		ñ =0.3	ñ =0.6	ñ =0.8	ñ =1		
0.1	0.2	0.009121	0.015739	0.017500	0.018491	0.018491	1.01E-09
0.2		0.018242	0.031479	0.035000	0.036982	0.036982	2.02E-09
0.3		0.027363	0.047218	0.052501	0.055472	0.055472	3.02E-09
0.4		0.036484	0.062957	0.070001	0.073963	0.073963	4.03E-09
0.5		0.045605	0.078697	0.087501	0.092454	0.092454	5.04E-09

0.1		0.004524	0.013383	0.015279	0.016524	0.016524	2.27E-07
0.2		0.009048	0.026766	0.030558	0.033048	0.033049	4.55E-07
0.3	0.5	0.013571	0.040149	0.045837	0.049572	0.049573	6.82E-07
0.4		0.018095	0.053532	0.061116	0.066096	0.066097	9.09E-07
0.5		0.022619	0.066915	0.076396	0.082620	0.082621	1.14E-06



(a) (b)

FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..4 COMPARISON OF SOLUTION $\Omega(\xi, \tau)$ FOR EXAMPLE-2



(a) (b)

FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT..5 SOLUTION $\Omega(\xi, \tau)$ FOR EXAMPLE-2 WITH VARIOUS

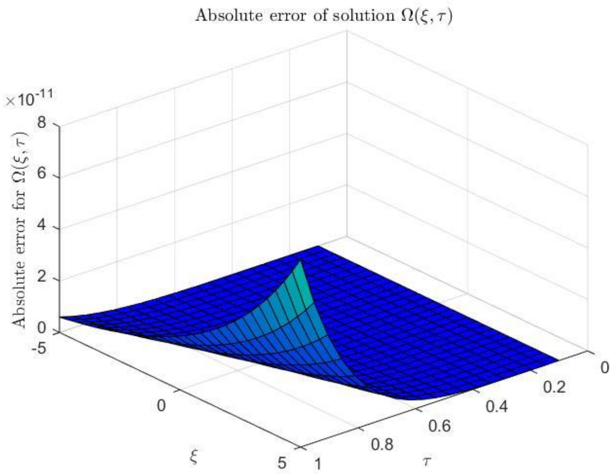


FIGURE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT.**.6**

TABLE ERROR! NO TEXT OF SPECIFIED STYLE IN DOCUMENT.**.2 NUMERICAL COMPARISON OF $\boldsymbol{\Omega(\xi,\tau)}$ FOR EXAMPLE-2 WITH VARIOUS FRACTIONAL ORDER $\boldsymbol{\varrho}$**

ξ	τ	CLADM				Exact	Abs error
		ñ =0.3	ñ =0.6	ñ =0.8	ñ =1		
0.1	0.2	0.291491	0.266699	0.261915	0.259553	0.259553	1.39E-15
0.2		0.298870	0.273450	0.268545	0.266124	0.266124	1.39E-15
0.3		0.306436	0.280373	0.275343	0.272861	0.272861	1.44E-15
0.4		0.314194	0.287471	0.282314	0.279768	0.279768	1.44E-15
0.5		0.322148	0.294748	0.289461	0.286850	0.286850	1.50E-15
0.1	0.5	0.303591	0.274564	0.268092	0.264466	0.264466	3.33E-13
0.2		0.311276	0.281515	0.274879	0.271161	0.271161	3.41E-13
0.3		0.319156	0.288641	0.281838	0.278025	0.278025	3.50E-13
0.4		0.327236	0.295948	0.288973	0.285063	0.285063	3.59E-13
0.5		0.335520	0.303440	0.296288	0.292280	0.292280	3.68E-13

5. Conclusion

This paper presented a rigorous formulation and validation of the Conformable Laplace Adomian Decomposition Method (CLADM) for the solution of nonlinear conformable fractional partial differential equations. The effectiveness and accuracy of the proposed method were assessed through two representative benchmark problems, namely the conformable fractional Damped Burgers equation and the conformable fractional reaction–diffusion equation. The obtained results demonstrate that CLADM yields rapidly convergent analytical series solutions with a high degree of accuracy. For Problem 1, errors were of order 10^{-9} at early times and 10^{-6} at later times when compared to exact solutions. Problem 2 achieved near-machine-precision accuracy ($\sim 10^{-15}$), confirming the method's numerical reliability. Both problems exhibited smooth behaviour across fractional orders $\tilde{n} \in [0.3, 1.0]$ without instabilities, validating the conformable derivative framework's robustness. These advantages position CLADM as an effective analytical tool for investigating fractional nonlinear PDEs.

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